AP Physics

Torque - Problem Set - Solutions

Mr. McMullen

1. Use the numbers from Example 8.1.

$$\tau = Fr_{\perp}, \quad \mathbf{\mathcal{F}} = \frac{\tau}{r_{\perp}} = \frac{18Nm}{(0.03m)\cos 30} = 693 \text{ N}$$

2.. When the force is applied perpendicular to the length of the wrench, minimum force is required and the lever arm equals the length of the wrench. At $\theta = 90^{\circ}$, $r_{\perp} = 0.15$ m.

$$\tau = Fr_{\perp}, \quad \mathbf{G} = \frac{\tau}{r_{\perp}} = \frac{25 \text{ m} \cdot \text{N}}{0.15 \text{ m}} = \frac{1.7 \times 10^2 \text{ N}}{1.7 \times 10^2 \text{ N}}.$$

3.. In this case, $r_{\perp} = (0.15 \text{ m}) \sin 30^{\circ}$. $F = \frac{\tau}{r_{\perp}} = \frac{25 \text{ m} \cdot \text{N}}{(0.15 \text{ m}) \sin 30^{\circ}} = \boxed{3.3 \times 10^2 \text{ N}}.$



4. 6 stable (faces), 20 unstable (12 edges and 8 corners).

so $x = \frac{m_1}{m_2} (2.0 \text{ m}) = \frac{35 \text{ kg}}{30 \text{ kg}} (2.0 \text{ m}) = \boxed{2.3 \text{ m}}.$

(a) Yes, the seesaw can be balanced if the lever arms are appropriate for the weights of the children, because torque is equal to force times the lever arm.
(b) Στ = 0, m₁ g(2.0 m) - m₂ g x = 0,

(a) $\Sigma \tau = 0$, (0.100 kg)g(0.500 m - 0.250 m) - (0.0750 kg)g(x - 0.500 m) = 0, so $x = 0.833 \text{ m} = \boxed{83.3 \text{ cm}}$.

(b) (0.100 kg)g(0.500 m - 0.250 m) - m(0.900 m - 0.500 m) = 0,

so
$$m = 0.0625 \text{ kg} = 62.5 \text{ g}$$
.

7..

(a) No, it is not possible to have the lines perfectly horizontal, because the weight has to be supported by an upward component of the tensions in the lines. If the lines were horizontal, then they cannot support the weight.

(b)
$$\theta = \tan^{-1}\left(\frac{0.010}{15}\right) = 0.0382^\circ$$
. $\Sigma F_y = 2T \sin\theta - mg = 0$.
So $T = \frac{mg}{2\sin\theta} = \frac{(0.25 \text{ kg})(9.80 \text{ m/s}^2)}{2\sin 0.0382^\circ} = 1.8 \times 10^3 \text{ N} > 400 \text{ lb}$.

8.

Choose the joint (where F_i is) as the axis of rotation. $\Sigma \tau = F_{\rm m} (0.18 \text{ m}) \sin 15^{\circ} - (3.0 \text{ kg})(9.80 \text{ m/s}^2)(0.26 \text{ m}) = 0, \quad \mathbf{\mathcal{F}}_{\rm m} = \boxed{1.6 \times 10^2 \text{ N}}$ 9. $\Sigma \tau = r_{\perp \mathrm{m}} F_{\mathrm{m}} - r_{\perp \mathrm{arm}} \left(m_{\mathrm{arm}} g \right) - r_{\perp \mathrm{b}} \left(m_{\mathrm{b}} g \right) = 0.$ $F_{\rm m} = \frac{r_{\perp \rm arm} (m_{\rm arm} g) + r_{\perp \rm b} (m_{\rm b} g)}{r_{\perp \rm m}} = \frac{(0.200 \text{ m})(8.50 \text{ kg})(9.80 \text{ m/s}^2) + (0.300 \text{ m})(5.00 \text{ kg})(9.80 \text{ m/s}^2)}{0.0400 \text{ m}}$ = 784 N.

10. (a) If the ball skids to your right, the frictional forced is to your left. Its torque is clockwise. (b) $f_k = \mu_k N = \mu_k mg = (0.400)(7.00 \text{ kg})(9.80 \text{ m/s}^2) = 27.44 \text{ N}.$ $\tau = Fr = (27.44 \text{ N})(0.170 \text{ m}) = 4.66 \text{ m} \cdot \text{N}.$

(a) The tension in the rope attached to m_1 is $T = (4.50 \text{ kg})(9.80 \text{ m/s}^2) = 44.1 \text{ N}$, and there are two such 11. tensions pulling the leg horizontally. So the reaction force is $R = 2T = \boxed{88.2 \text{ N}}$. (b) $\Sigma F_{\rm y} = 0$, $\sigma = (m_1 + m_2)g - Mg = 0$, so $m_2 = M - m_1 = 15.0 \text{ kg} - 4.50 \text{ kg} = 10.5 \text{ kg}$. $\tau = Fr_{\perp} = mg (r \sin \theta) = mgr \sin \theta = (5.0 \text{ kg})(9.80 \text{ m/s}^2)(0.400 \text{ m}) \sin \theta = (19.6 \text{ m} \cdot \text{N}) \sin \theta$ 37.

$$0^{\circ}: \quad \tau = (19.6 \text{ m}\cdot\text{N}) \sin 0^{\circ} = \boxed{0}; \qquad 30^{\circ}: \quad \tau = (19.6 \text{ m}\cdot\text{N}) \sin 30^{\circ} = \boxed{9.80 \text{ m}\cdot\text{N}}; \\ 60^{\circ}: \quad \tau = (19.6 \text{ m}\cdot\text{N}) \sin 60^{\circ} = \boxed{17.0 \text{ m}\cdot\text{N}}; \qquad 90^{\circ}: \quad \tau = (19.6 \text{ m}\cdot\text{N}) \sin 90^{\circ} = \boxed{19.6 \text{ m}\cdot\text{N}}.$$

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12.
           Choose where the string is as the axis and work from the bottom up. Apply \Sigma \tau = 0 to the
                                                m_1 g(40 \text{ cm}) - m_2 g(20 \text{ cm}) = 0, \quad \mathbf{r} = m_2 = 2m_1 = \boxed{0.20 \text{ kg}}.
           bees:
           bees-1st bird combination: (m_1 + m_2)g(25 \text{ cm}) - m_3 g(15 \text{ cm}), \quad @ m_3 = \frac{5}{3} (m_1 + m_2) = \boxed{0.50 \text{ kg}}.
           bees and 1st bird–2nd bird combination: m_4 g(30 \text{ cm}) - (m_1 + m_2 + m_3)g(15 \text{ cm}) = 0,
           so m_4 = \frac{1}{2}(m_1 + m_2 + m_3) = 0.40 \text{ kg}.
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(a) The location of the center of gravity is expected to be (2) toward the scale at the person's head, 14. because the mass distribution of the human body is more toward the upper body than the lower body. (b) Choose the center of gravity as the axis of rotation. $\Sigma \tau = (25 \text{ kg})gx - (30 \text{ kg})g(1.6 \text{ m} - \text{x}) = 0$, or 25x - 48 + 30x = 0. Solving, x = 0.87 m from the feet.

15. (a) The center of gravity (CG) of the first book is at the center. For the last book not to fall, its CG can not displace more than (25.0 cm)/2 = 12.5 cm relative to the CG of the first book (within the base). The CG of each successive book on the top is moved (3.00 cm)/2 = 1.5 cm relative to that of the one below. The number of books on top of the first one can be $\frac{12.5 \text{ cm}}{1.5 \text{ cm}} = 8.33$. Thus the total is 1 + 8 = 9 books, including the first one.

(b) The total height is 9(5.0) cm = 45 cm. So the center of mass (CM) is at $\frac{45 \text{ cm}}{2} = 22.5 \text{ cm}$.

- 16. Yes. The center of gravity of every stick is at or to the left of the edge of the table.
- 17. The height of the center of gravity at stable equilibrium is

$$d = \frac{0.500 \text{ m}}{2} = 0.250 \text{ m}.$$

The minimum height of the center of gravity at unstable equilibrium is half the diagonal distance

$$d' = \sqrt{2} \ \frac{0.500 \text{ m}}{2} = 0.3536 \text{ m}.$$

So the minimum distance that CG has to be raised is 0.3536 m - 0.250 m = 0.1036 m.

Therefore the work done against gravity is $W = (10.0 \text{ kg})(9.80 \text{ m/s}^2)(0.1036 \text{ m}) = 10.2 \text{ J}$.

18. When it is about to tip over the left support, the force on the right support is zero.Choose the left support as the axis of rotation.

 $\Sigma \tau = 0$, **G** (70 kg)g x - (15 kg)g(1.25 m) = 0,

so x = 0.27 m.

Therefore it is 1.5 m - 0.27 m = 1.2 m from left end of board.

19. Choose the left end as the axis. $\Sigma \tau = 0$,

 $T_2(0) - (70 \text{ kg})(9.80 \text{ m/s}^2)(1.5 \text{ m}) - (15 \text{ kg})(9.80 \text{ m/s}^2)(2.75 \text{ m}) + T_1(5.5 \text{ m}) = 0,$

so
$$T_1 = \lfloor 2.6 \times 10^2 \text{ N} \rfloor$$
.
 $\Sigma F_y = 0$, $T_1 + T_2 = (70 \text{ kg} + 15 \text{ kg})(9.80 \text{ m/s}^2)$,
so $T_2 = 833 \text{ N} - 261 \text{ N} = \boxed{5.7 \times 10^2 \text{ N}}$.

 T_2 can also be found by choosing the right end as the axis.





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20.
$$\Sigma F_x = -T_1 \cos 45^\circ + T_2 \cos 30^\circ = 0$$
, or $-\sqrt{2} T_1 + \sqrt{3} T_2 = 0$ (1)

$$\Sigma F_{y} = T_{1} \sin 45^{\circ} - T_{2} \sin 30^{\circ} - mg = 0, \quad \text{or} \quad \sqrt{2} \ T_{1} - T_{2} - 2mg = 0.$$
(2)
Equation (1) + Equation (2) gives $(\sqrt{3} - 1)T_{2} = 2mg$,

so
$$T_2 = \frac{2(1.5 \text{ kg})(9.80 \text{ m/s}^2)}{\sqrt{3} - 1} = \boxed{40 \text{ N}}$$
 and $T_1 = \sqrt{\frac{3}{2}} T_2 = \boxed{49 \text{ N}}.$

21`.
$$\Sigma F_y = T_1 \sin 45^\circ - mg = 0$$
, $T_1 = \frac{mg}{\sin 45^\circ} = \frac{(1.5 \text{ kg})(9.80 \text{ m/s}^2)}{\sin 45^\circ} = 21 \text{ N}$.
 $\Sigma F_x = -T_1 \cos 45^\circ + T_2 = 0$, $T_2 = T_1 \cos 45^\circ = 15 \text{ N}$.