SECTION 1 Planetary Motion and Gravitation

Our solar system includes the Sun, Earth and seven other major planets, dwarf planets, and interplanetary dust and gas. Various moons orbit the planets. What holds all this together?

MAIN IDEA

The gravitational force between two objects is proportional to the product of their masses divided by the square of the distance between them.

Essential Questions

- What is the relationship between a planet's orbital radius and period?
- What is Newton's law of universal gravitation, and how does it relate to Kepler's laws?
- Why was Cavendish's investigation important?

Review Vocabulary

Newton's third law states all forces come in pairs and that the two forces in a pair act on different objects, are equal in strength, and are opposite in direction

New Vocabulary

Kepler's first law
Kepler's second law
Kepler's third law
gravitational force
law of universal gravitation

Early Observations

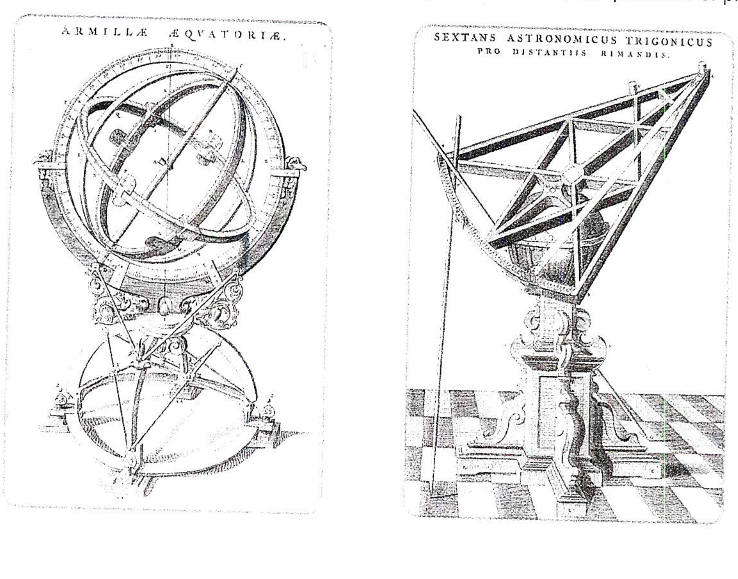
In ancient times, the Sun, the Moon, the planets, and the stars were assumed to revolve around Earth. Nicholas Copernicus, a Polish astronomer, noticed that the best available observations of the movements of planets did not fully agree with the Earth-centered model.

The results of his many years of work were published in 1543, when Copernicus was on his deathbed. His book showed that the motion of planets is much more easily understood by assuming that Earth and other planets revolve around the Sun. His model helped explain phenomena such as the inner planets Mercury and Venus always appearing near the Sun. Copernicus's view advanced our understanding of planetary motion. He incorrectly assumed, however, that planetary orbits are circular. This assumption did not fit well with observations, and modification of Copernicus's model was necessary to make it accurate.

Tycho Brahe was born a few years after Copernicus died. As a boy of 14 in Denmark, Tycho observed an eclipse of the Sun on August 21, 1560. The fact that it had been predicted inspired him toward a career in astronomy.

As Tycho studied astronomy, he realized that the charts of the time did not accurately predict astronomical events. Tycho recognized that measurements were required from one location over a long period of time. He was granted an estate on the Danish island of Hven and the funding to build an early research institute. Telescopes had not been invented, so to make measurements, Tycho used huge instruments that he designed and built in his own shop, such as those shown in **Figure 1**. Tycho is credited with the most accurate measurements of the time.

Figure 1 Instruments such as these were used by Tycho to measure the positions of planets.



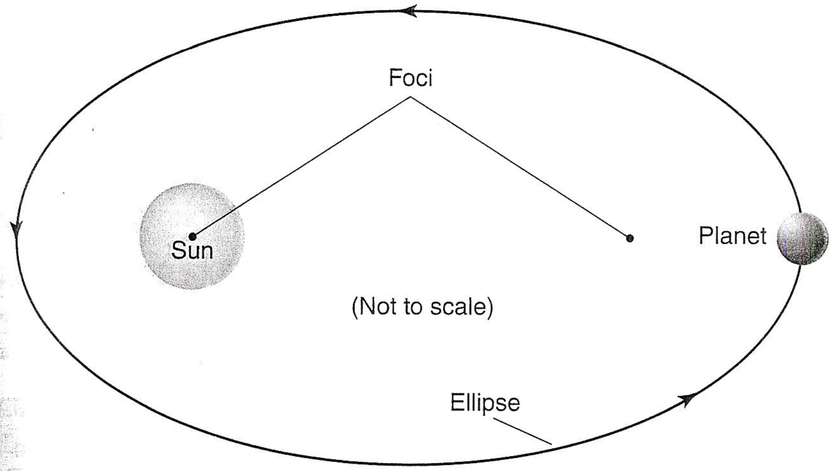


Figure 2 The orbit of each planet is an ellipse, with the Sun at one focus.

Kepler's Laws

In 1600 Tycho moved to Prague where Johannes Kepler, a 29-year-old German, became one of his assistants. Kepler analyzed Tycho's observations. After Tycho's death in 1601, Kepler continued to study Tycho's data and used geometry and mathematics to explain the motion of the planets. After seven years of careful analysis of Tycho's data on Mars, Kepler discovered the laws that describe the motion of every planet and satellite, natural or artificial. Here, the laws are presented in terms of planets.

Kepler's first law states that the paths of the planets are ellipses, with the Sun at one focus. An ellipse has two foci, as shown in **Figure 2.** Although exaggerated ellipses are used in the diagrams, Earth's actual orbit is very nearly circular. You would not be able to distinguish it from a circle visually.

View an **animation of Kepler's first law.**

Concepts In Motion

Kepler found that the planets move faster when they are closer to the Sun and slower when they are farther away from the Sun. **Kepler's second law** states that an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals, as illustrated in **Figure 3.**

READING CHECK Compare the distances traveled from point 1 to point 2 and from point 6 to point 7 in **Figure 3.** Through which distance would Earth be traveling fastest?

A period is the time it takes for one revolution of an orbiting body. Kepler also discovered a mathematical relationship between periods of planets and their mean distances away from the Sun.

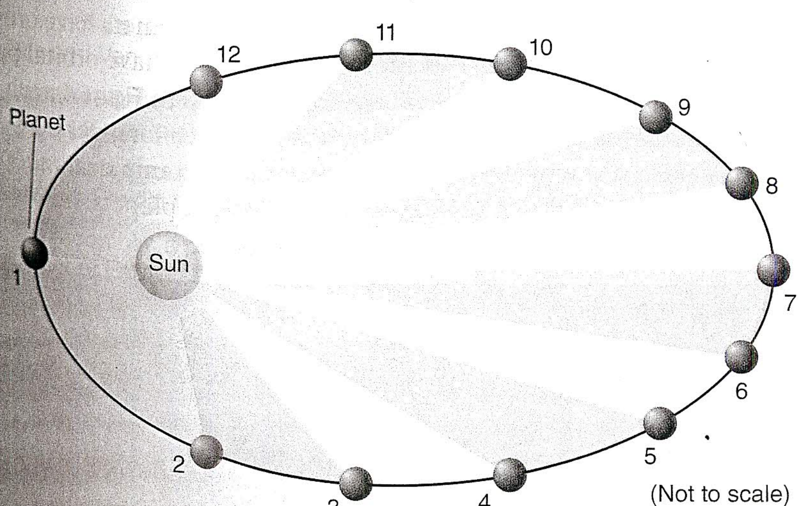


Figure 3 Kepler found that elliptical orbits sweep out equal areas in equal time periods.

Explain why the equal time areas are shaped differently.

View an **animation and a simulation of Kepler's second law.**

Concepts In Motion

Table 1 Solar System Data

Name	Average Radius (m)	Mass (kg)	Average Distance from the Sun (m)
Sun	6.96×10^{8}	1.99×10^{30}	—
Mercury	2.44×10^{6}	3.30×10^{23}	5.79×10^{10}
Venus	6.05×10^{6}	4.87×10^{24}	1.08×10^{11}
Earth	6.38×10^{6}	5.97×10^{24}	1.50×10^{11}
Mars	3.40×10^{6}	6.42×10^{23}	2.28×10^{11}
Jupiter	7.15×10^{7}	1.90×10^{27}	7.79×10^{11}
Saturn	6.03×10^{7}	5.68×10^{26}	1.43×10^{12}
Uranus	2.56×10^{7}	8.68×10^{25}	2.87×10^{12}
Neptune	2.48×10^{7}	1.02×10^{26}	4.50×10^{12}

View an **animation and a simulation of Kepler's third law.**

Concepts in Motion

Kepler's third law states that the square of the ratio of the periods of any two planets revolving about the Sun is equal to the cube of the ratio of their average distances from the Sun. Thus, if the periods of the planets are T_A and T_B and their average distances from the Sun are r_A and r_B, Kepler's third law can be expressed as follows.

KEPLER'S THIRD LAW

The square of the ratio of the period of planet A to the period of planet B is equal to the cube of the ratio of planet A's average distance from the Sun to planet B's average distance from the Sun.

$$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$$

Note that Kepler's first two laws apply to each planet, moon, and satellite individually. The third law, however, relates the motion of two objects around a single body. For example, it can be used to compare the planets' distances from the Sun, shown in **Table 1,** to their periods around the Sun. It also can be used to compare distances and periods of the Moon and artificial satellites orbiting Earth.

PhysicsLAB

MODELING ORBITS

What is the shape of the orbits of planets and satellites in the solar system?

PhysicsLAB

Comet periods Comets are classified as long-period comets or short-period comets based on orbital periods. Long-period comets have orbital periods longer than 200 years and short-period comets have orbital periods shorter than 200 years. Comet Hale-Bopp, shown in **Figure 4,** with a period of approximately 2400 years, is an example of a long-period comet. Comet Halley, with a period of 76 years, is an example of a short-period comet. Comets also obey Kepler's laws. Unlike planets, however, comets have highly elliptical orbits.

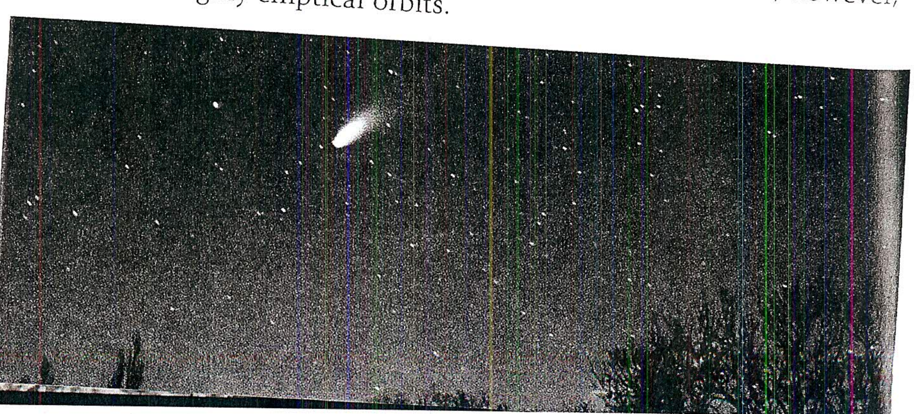
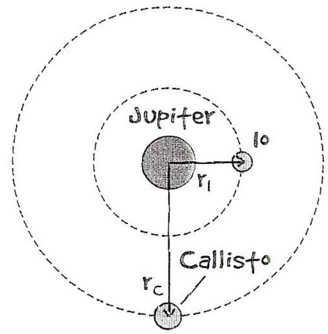


Figure 4 Hale-Bopp is a long-period comet, with a period of 2400 years. This photo was taken in 1997, when Hale-Bopp was highly visible.

CALLISTO'S DISTANCE FROM JUPITER Galileo measured the orbital radii of Jupiter's moons using the diameter of Jupiter as a unit of measure. He found that Io, the closest moon to Jupiter, has a period of 1.8 days and is 4.2 units from the center of Jupiter. Callisto, the fourth moon from Jupiter, has a period of 16.7 days. Using the same units that Galileo used, predict Callisto's distance from Jupiter.



1 ANALYZE AND SKETCH THE PROBLEM

- Sketch the orbits of Io and Callisto.
- Label the radii.

KNOWN	UNKNOWN
$T_C = 16.7$ days	$r_C = ?$
$T_I = 1.8$ days	
$r_I = 4.2$ units	

2 SOLVE FOR CALLISTO'S DISTANCE FROM JUPITER

Solve Kepler's third law for r_C.

$$\left(\frac{T_C}{T_I}\right)^2 = \left(\frac{r_C}{r_I}\right)^3$$

$$r_C{}^3 = r_I{}^3\left(\frac{T_C}{T_I}\right)^2$$

$$r_C = \sqrt[3]{r_I{}^3\left(\frac{T_C}{T_I}\right)^2}$$

◀ Substitute $r_I = 4.2$ units, $T_C = 16.7$ days, $T_I = 1.8$ days

$$= \sqrt[3]{(4.2\text{ units})^3\left(\frac{16.7\text{ days}}{1.8\text{ days}}\right)^2}$$

$$= \sqrt[3]{6.4\times10^3\text{ units}^3}$$

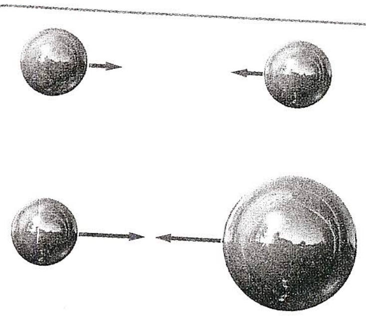
$$= 19\text{ units}$$

3 EVALUATE THE ANSWER

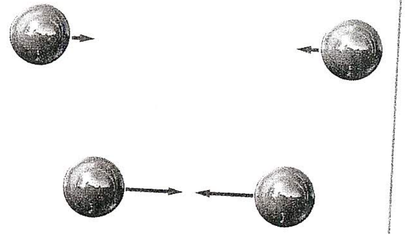
- **Are the units correct?** r_C should be in Galileo's units, like r_I.
- **Is the magnitude realistic?** The period is larger, so the radius should be larger.

PRACTICE PROBLEMS

1. If Ganymede, one of Jupiter's moons, has a period of 32 days, how many units is its orbital radius? Use the information given in Example Problem 1.
2. An asteroid revolves around the Sun with a mean orbital radius twice that of Earth's. Predict the period of the asteroid in Earth years.
3. Venus has a period of revolution of 225 Earth days. Find the distance between the Sun and Venus as a multiple of Earth's average distance from the Sun.
4. Uranus requires 84 years to circle the Sun. Find Uranus's average distance from the Sun as a multiple of Earth's average distance from the Sun.
5. From **Table 1** you can find that, on average, Mars is 1.52 times as far from the Sun as Earth is. Predict the time required for Mars to orbit the Sun in Earth days.
6. The Moon has a period of 27.3 days and a mean distance of 3.90×10^5 km from the center of Earth.
   a. Use Kepler's laws to find the period of a satellite in orbit 6.70×10^3 km from the center of Earth.
   b. How far above Earth's surface is this satellite?
7. **CHALLENGE** Using the data in the previous problem for the period and radius of revolution of the Moon, predict what the mean distance from Earth's center would be for an artificial satellite that has a period of exactly 1.00 day.



Gravitation between objects depends on the product of the masses of the objects.



The gravitational force between objects is inversely proportional to the square of the distance between the objects.

Figure 5 Mass and distance affect the magnitude of the gravitational force between objects.

View an **animation of the law of universal gravitation.**

Concepts In Motion

Newton's Law of Universal Gravitation

In 1666, Isaac Newton began his studies of planetary motion. It has been said that seeing an apple fall made Newton wonder if the force that caused the apple to fall might extend to the Moon, or even beyond. He found that the magnitude of the force (F) on a planet due to the Sun varies inversely with the square of the distance (r) between the centers of the planet and the Sun. That is, F is proportional to $\frac{1}{r^2}$. The force (F) acts in the direction of the line connecting the centers of the two objects, as shown in **Figure 5.**

Newton found that both the apple's and the Moon's accelerations agree with the $\frac{1}{r^2}$ relationship. According to his own third law, the force Earth exerts on the apple is exactly the same as the force the apple exerts on Earth. Even though these forces are exactly the same, you can easily observe the effect of the force on the apple because it has much lower mass than Earth. The force of attraction between two objects must be proportional to the objects' masses and is known as the **gravitational force.**

Newton was confident that the same force of attraction would act between any two objects anywhere in the universe. He proposed the **law of universal gravitation,** which states that objects attract other objects with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them as shown below.

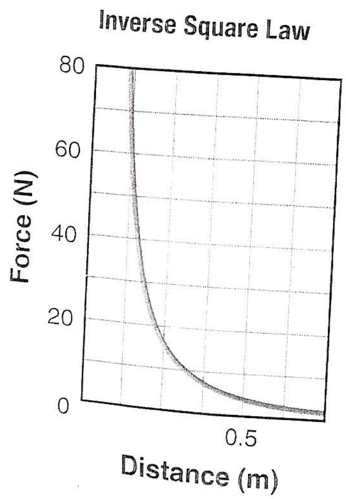
LAW OF UNIVERSAL GRAVITATION

The gravitational force is equal to the universal gravitational constant, times the mass of object 1, times the mass of object 2, divided by the distance between the centers of the objects, squared.

$$F = \frac{Gm_1m_2}{r^2}$$

According to Newton's equation, F is directly proportional to m_1 and m_2. If the mass of a planet near the Sun doubles, the force of attraction doubles. Use the Connecting Math to Physics feature below to examine how changing one variable affects another. **Figure 6** illustrates the inverse square relationship graphically. The term G is the universal gravitational constant and will be discussed in the next sections.

Figure 6 This is a graphical representation of the inverse square relationship.



CONNECTING MATH

Direct and Inverse Relationships Newton's law of universal gravitation has both direct and inverse relationships.

$F \propto m_1m_2$		$F \propto \frac{1}{r^2}$	
Change	**Result**	**Change**	**Result**
$(2m_1)m_2$	$2F$	$2r$	$\frac{1}{4}F$
$(3m_1)m_2$	$3F$	$3r$	$\frac{1}{9}F$
$(2m_1)(3m_2)$	$6F$	$\frac{1}{2}r$	$4F$
$\left(\frac{1}{2}\right)m_1m_2$	$\frac{1}{2}F$	$\frac{1}{3}r$	$9F$

Universal Gravitation and Kepler's Third Law

Newton stated the law of universal gravitation in terms that applied to the motion of planets about the Sun. This agreed with Kepler's third law and confirmed that Newton's law fit the best observations of the day.

Consider a planet orbiting the Sun, as shown in **Figure 7.** Newton's second law of motion, $F_{net} = ma$, can be written as $F_{net} = m_P a_c$, where F_{net} is the magnitude of the gravitational force, m_P is the mass of the planet, and a_c is the centripetal acceleration of the planet. For simplicity, assume circular orbits. Recall from your study of uniform circular motion that for a circular orbit $a_c = \frac{4\pi^2 r}{T^2}$. This means that $F_{net} = m_P a_c$ may now be written $F_{net} = \frac{m_P 4\pi^2 r}{T^2}$. In this equation, T is the time in seconds required for the planet to make one complete revolution about the Sun. If you set the right side of this equation equal to the right side of the law of universal gravitation, you arrive at the following result:

$$\frac{Gm_S m_P}{r^2} = \frac{m_P 4\pi^2 r}{T^2}$$

$$T^2 = \left(\frac{4\pi^2}{Gm_S}\right) r^3$$

$$T = \sqrt{\left(\frac{4\pi^2}{Gm_S}\right) r^3}$$

The period of a planet orbiting the Sun can be expressed as follows.

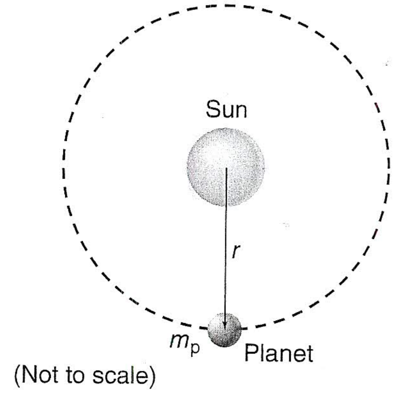


Figure 7 A planet with mass m_P and average distance from the Sun r orbits the Sun. The mass of the Sun is m_S.

PERIOD OF A PLANET ORBITING THE SUN

The period of a planet orbiting the Sun is equal to 2π times the square root of the average distance from the Sun cubed, divided by the product of the universal gravitational constant and the mass of the Sun.

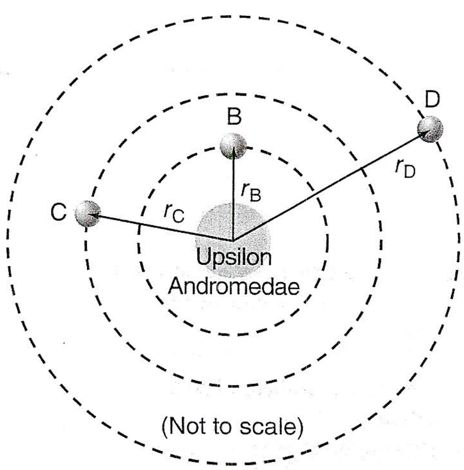
$$T = 2\pi\sqrt{\frac{r^3}{Gm_S}}$$

Squaring both sides makes it apparent that this equation is Kepler's third law of planetary motion: the square of the period is proportional to the cube of the distance that separates the masses. The factor $\frac{4\pi^2}{Gm_S}$ depends on the mass of the Sun and the universal gravitational constant. Newton found that this factor applied to elliptical orbits as well.

PHYSICS CHALLENGE

Astronomers have detected three planets that orbit the star Upsilon Andromedae. Planet B has an average orbital radius of 0.0595 AU and a period of 4.6171 days. Planet C has an average orbital radius of 0.832 AU and a period of 241.33 days. Planet D has an average orbital radius of 2.53 AU and a period of 1278.1 days. (Distances are given in astronomical units (AU)—Earth's average distance from the Sun. The distance from Earth to the Sun is 1.00 AU.)

1. Do these planets obey Kepler's third law?
2. Find the mass of the star Upsilon Andromedae in units of the Sun's mass. *Hint: compare $\frac{r^3}{T^2}$ for these planets with that of Earth in the same units (AU and days).*



Measuring the Universal Gravitational Constant

How large is the constant G? As you know, the force of gravitational attraction between two objects on Earth is relatively small. The slightest attraction, even between two massive bowling balls, is almost impossible to detect. In fact, it took 100 years from the time of Newton's work for scientists to develop an apparatus that was sensitive enough to measure the force of gravitational attraction.

Cavendish's apparatus In 1798 English scientist Henry Cavendish used equipment similar to the apparatus shown in **Figure 8** to measure the gravitational force between two objects. The apparatus has a horizontal rod with small lead spheres attached to each end. The rod is suspended at its midpoint so that it can rotate. Because the rod is suspended by a thin wire, the rod and spheres are very sensitive to horizontal forces.

To measure G, two large spheres are placed in a fixed position close to each of the two small spheres, as shown in **Figure 8.** The force of attraction between the large and small spheres causes the rod to rotate. When the force required to twist the wire equals the gravitational force between the spheres, the rod stops rotating. By measuring the angle through which the rod turns, the attractive force between the objects can be calculated.

Investigate **universal gravitation.**

Virtual Investigation

READING CHECK **Explain** why the rod and sphere in Cavendish's apparatus must be sensitive to horizontal forces.

The angle through which the rod turns is measured by using a beam of light that is reflected from the mirror. The distances between the sphere's centers and the force can both be measured. The masses of the spheres are known. By substituting the values for force, mass, and distance into Newton's law of universal gravitation, an experimental value for G is found: when m_1 and m_2 are measured in kilograms, r in meters, and F in newtons, $G = 6.67 \times 10^{-11}\ N \cdot m^2/kg^2$.

Cavendish Balance

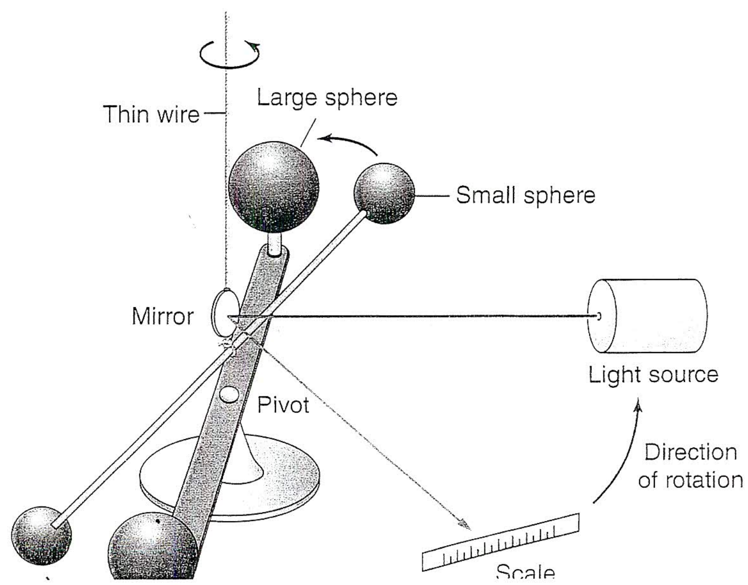


Figure 8 A Cavendish balance uses a light source and a mirror to measure the movement of the spheres.

View an **animation of Cavendish's**

The Importance of G Cavendish's investigation often is called "weighing Earth" because it helped determine Earth's mass. Once the value of G is known, not only the mass of Earth, but also the mass of the Sun can be determined. In addition, the gravitational force between any two objects can be calculated by using Newton's law of universal gravitation. For example, the attractive gravitational force (F_g) between two bowling balls of mass 7.26 kg, with their centers separated by 0.30 m, can be calculated as follows:

$$F_g = \frac{(6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2)(7.26\ \text{kg})(7.26\ \text{kg})}{(0.30\ \text{m})^2} = 3.9\times10^{-8}\ \text{N}$$

You know that on Earth's surface, the weight of an object of mass m is a measure of Earth's gravitational attraction: $F_g = mg$. If Earth's mass is represented by m_E and Earth's radius is represented by r_E, the following is true:

$$F_g = \frac{Gm_Em}{{r_E}^2} = mg, \text{ and so } g = \frac{Gm_E}{{r_E}^2}$$

This equation can be rearranged to solve for m_E.

$$m_E = \frac{g{r_E}^2}{G}$$

Using $g = 9.8$ N/kg, $r_E = 6.38\times10^6$ m, and $G = 6.67\times10^{-11}$ N·m²/kg², the following result is obtained for Earth's mass:

$$m_E = \frac{(9.8\ \text{N/kg})(6.38\times10^6\ \text{m})^2}{6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2} = 5.98\times10^{24}\ \text{kg}$$

When you compare the mass of Earth to that of a bowling ball, you can see why the gravitational attraction between everyday objects is not easily observed. Cavendish's investigation determined the value of G, confirmed Newton's prediction that a gravitational force exists between any two objects, and helped calculate the mass of Earth **(Figure 9).**

Figure 9 Cavendish's investigations helped calculate the mass of Earth.

SECTION 1 REVIEW

Check your understanding.

8. **MAINIDEA** What is the gravitational force between two 15-kg balls whose centers are 35 m apart? What fraction is this of the weight of one ball?

9. **Neptune's Orbital Period** Neptune orbits the Sun at an average distance given in **Figure 10,** which allows gases, such as methane, to condense and form an atmosphere. If the mass of the Sun is 1.99×10^{30} kg, calculate the period of Neptune's orbit.

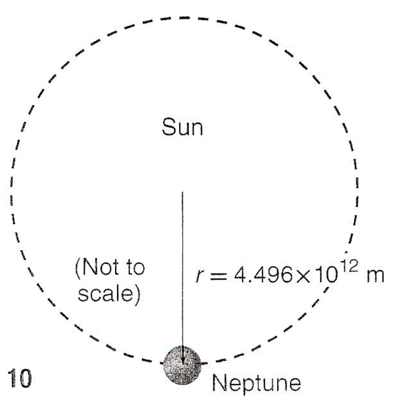


Figure 10

10. **Gravity** If Earth began to shrink, but its mass remained the same, what would happen to the value of g on Earth's surface?

11. **Universal Gravitational Constant** Cavendish did his investigation using lead spheres. Suppose he had replaced the lead spheres with copper spheres of equal mass. Would his value of G be the same or different? Explain.

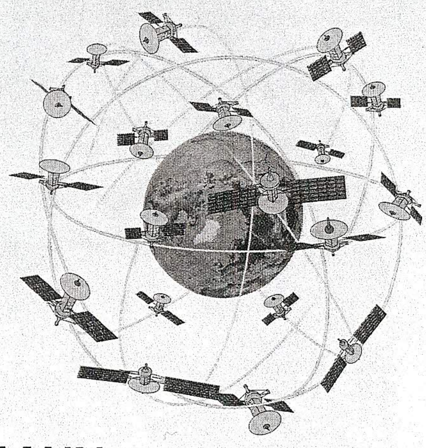
12. Kepler's three statements and Newton's equation for gravitational attraction are called laws. Were they ever theories? Will they ever become theories?

13. **Critical Thinking** Picking up a rock requires less effort on the Moon than on Earth.

    a. How will the Moon's gravitational force affect the path of the rock if it is thrown horizontally?

    b. If the thrower accidentally drops the rock on her toe, will it hurt more or less than it would on Earth? Explain.

SECTION 2 Using the Law of Universal Gravitation



MAIN IDEA

All objects are surrounded by a gravitational field that affects the motions of other objects.

Essential Questions

- How can you describe orbital motion?
- How are gravitational mass and inertial mass alike, and how are they different?
- How is gravitational force explained, and what did Einstein propose about gravitational force?

Review Vocabulary

centripetal acceleration the center-seeking acceleration of an object moving in a circle at a constant speed

New Vocabulary

inertial mass
gravitational mass

PHYSICS 4 YOU Have you ever used a device to locate your position or to map where you want to go? Where does that device get information? The Global Positioning System (GPS) consists of many satellites circling Earth. GPS satellites give accurate position data anywhere on or near Earth.

Orbits of Planets and Satellites

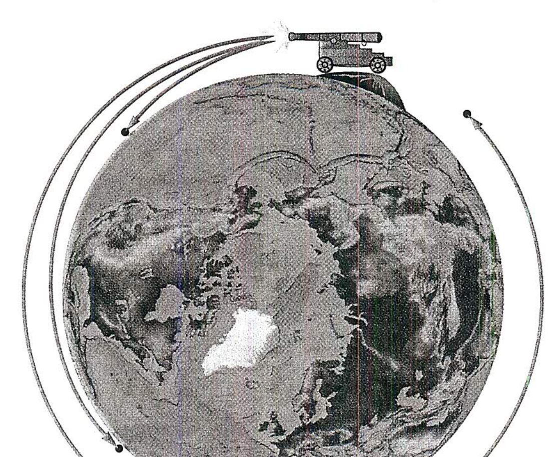
The planet Uranus was discovered in 1781. By 1830 it was clear that the law of gravitation didn't correctly predict its orbit. Two astronomers proposed that Uranus was being attracted by the Sun and by an undiscovered planet. They calculated the orbit of such a planet in 1845, and, one year later, astronomers at the Berlin Observatory found the planet now called Neptune. How is it possible for planets, such as Neptune and Uranus, to remain in orbit around the Sun?

Newton used a drawing similar to the one shown in **Figure 11** to illustrate a thought experiment on the motion of satellites. Imagine a cannon, perched high atop a mountain, firing a cannonball horizontally with a given horizontal speed. The cannonball is a projectile, and its motion has both vertical and horizontal components. Like all projectiles on Earth, it would follow a parabolic trajectory and fall back to the ground.

If the cannonball's horizontal speed were increased, it would travel farther across the surface of Earth and still fall back to the ground. If an extremely powerful cannon were used, however, the cannonball would travel all the way around Earth and keep going. It would fall toward Earth at the same rate that Earth's surface curves away. In other words, the curvature of the projectile would continue to just match the curvature of Earth so that the cannonball would never get any closer to or farther away from Earth's curved surface. The cannonball would, therefore, be in orbit.

Figure 11 Newton imagined a projectile launched parallel to Earth. If it has enough speed it will fall toward Earth with a curvature that matches the curvature of Earth's surface.

Identify the factor that is not considered in this example.



Investigate **Newton's cannon.**

Virtual Investigation

Newton's thought experiment ignored air resistance. For the cannonball to be free of air resistance, the mountain on which the cannon is perched would have to be more than 150 km above Earth's surface. By way of comparison, the mountain would have to be much taller than the peak of Mount Everest, the world's tallest mountain, which is only 8.85 km in height. A cannonball launched from a mountain that is 150 km above Earth's surface would encounter little or no air resistance at an altitude of 150 km because the mountain would be above most of the atmosphere. Thus, a cannonball or any object or satellite at or above this altitude could orbit Earth.

A satellite's speed A satellite in an orbit that is always the same height above Earth moves in uniform circular motion. Recall that its centripetal acceleration is given by $a_c = \frac{v^2}{r}$. Newton's second law, $F_{net} = ma_c$, can thus be rewritten $F_{net} = \frac{mv^2}{r}$. If Earth's mass is m_E, then this expression combined with Newton's law of universal gravitation produces the following equation:

$$\frac{Gm_E m}{r^2} = \frac{mv^2}{r}$$

Solving for the speed of a satellite in circular orbit about Earth (v) yields the following.

SPEED OF A SATELLITE ORBITING EARTH

The speed of a satellite orbiting Earth is equal to the square root of the universal gravitational constant times the mass of Earth, divided by the radius of the orbit.

$$v = \sqrt{\frac{Gm_E}{r}}$$

A satellite's orbital period A satellite's orbit around Earth is similar to a planet's orbit about the Sun. Recall that the period of a planet orbiting the Sun is expressed by the following equation:

$$T = 2\pi\sqrt{\frac{r^3}{Gm_S}}$$

Thus, the period for a satellite orbiting Earth is given by the following equation.

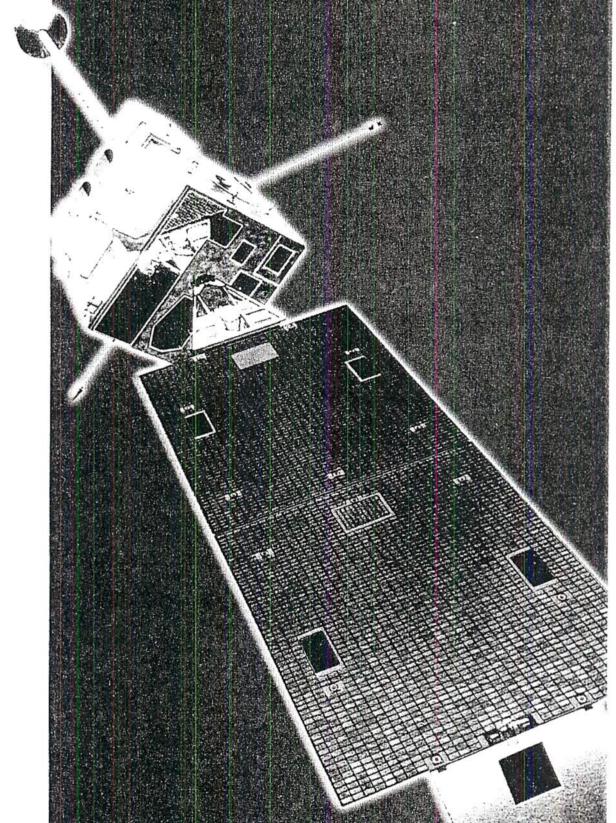
PERIOD OF A SATELLITE ORBITING EARTH

The period for a satellite orbiting Earth is equal to 2π times the square root of the radius of the orbit cubed, divided by the product of the universal gravitational constant and the mass of Earth.

$$T = 2\pi\sqrt{\frac{r^3}{Gm_E}}$$

The equations for the speed and period of a satellite can be used for any object in orbit about another. The mass of the central body will replace m_E in the equations, and r will be the distance between the centers of the orbiting body and the central body. Orbital speed (v) and period (T) are independent of the mass of the satellite.

☑ READING CHECK **Describe** how the mass of a satellite affects that satellite's orbital speed and period.



REAL-WORLD PHYSICS

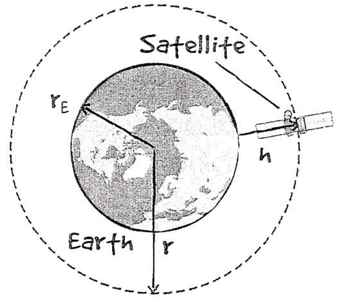
GEOSYNCHRONOUS ORBIT The GOES weather satellites orbit Earth once a day at an altitude of 35,785 km. The orbital speed of the satellite matches Earth's rate of rotation. Thus, to an observer on Earth, the satellite appears to remain above one spot. Dish antennas on Earth can be directed to one point in the sky and remain in a fixed position as the satellite orbits.

EXAMPLE PROBLEM 2

ORBITAL SPEED AND PERIOD Assume that a satellite orbits Earth 225 km above its surface. Given that the mass of Earth is 5.97×10^{24} kg and the radius of Earth is 6.38×10^{6} m, what are the satellite's orbital speed and period?

1 ANALYZE AND SKETCH THE PROBLEM

Sketch the situation showing the height of the satellite's orbit.



KNOWN	UNKNOWN
$h = 2.25\times10^{5}$ m	$v = ?$
$r_E = 56.38\times10^{6}$ m	$T = ?$
$m_E = 5.97\times10^{24}$ kg	
$G = 6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2$	

2 SOLVE FOR ORBITAL SPEED AND PERIOD

Determine the orbital radius by adding the height of the satellite's orbit to Earth's radius.

$$r = h + r_E$$
$$= 2.25\times10^{5}\ \text{m} + 6.38\times10^{6}\ \text{m} = 6.61\times10^{6}\ \text{m}$$

◀ Substitute $h = 2.25\times10^{5}$ m, $r_E = 6.38\times10^{6}$ m.

Solve for the speed.

$$v = \sqrt{\frac{Gm_E}{r}}$$
$$= \sqrt{\frac{(6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{6.61\times10^{6}\ \text{m}}}$$
$$= 7.76\times10^{3}\ \text{m/s}$$

◀ Substitute $G = 6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2$, $m_E = 5.97\times10^{24}$ kg, $r = 6.61\times10^{6}$ m.

Solve for the period.

$$T = 2\pi\sqrt{\frac{r^3}{Gm_E}}$$
$$= 2\pi\sqrt{\frac{(6.61\times10^{6}\ \text{m})^3}{(6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}}$$
$$= 5.35\times10^{3}\ \text{s}$$

◀ Substitute $r = 6.61\times10^{6}$ m, $G = 6.67\times10^{-11}\ \text{N·m}^2/\text{kg}^2$, $m_E = 5.97\times10^{24}$ kg

This is approximately 89 min, or 1.5 h.

3 EVALUATE THE ANSWER

Are the units correct? The unit for speed is meters per second, and the unit for period is seconds.

PRACTICE PROBLEMS

For the following problems, assume a circular orbit for all calculations.

14. Suppose that the satellite in Example Problem 2 is moved to an orbit that is 24 km larger in radius than its previous orbit.

a. What is its speed?

b. Is this faster or slower than its previous speed?

c. Why do you think this is so?

15. Uranus has 27 known moons. One of these moons is Miranda, which orbits at a radius of 1.29×10^{8} m. Uranus has a mass of 8.68×10^{25} kg. Find the orbital speed of Miranda. How many Earth days does it take Miranda to complete one orbit?

16. Use Newton's thought experiment on the motion of satellites to solve the following.

a. Calculate the speed that a satellite shot from a cannon must have to orbit Earth 150 km above its surface.

b. How long, in seconds and minutes, would it take for the satellite to complete one orbit and return to the cannon?

17. CHALLENGE Use the data for Mercury in **Table 1** to find the following.

a. the speed of a satellite that is in orbit 260 km above Mercury's surface

b. the period of the satellite

▸ **CONNECTION TO EARTH SCIENCE** *Landsat 7,* shown in **Figure 12,** is an artificial satellite that provides images of Earth's continental surfaces. *Landsat 7* images have been used to create maps, study land use, and monitor resources and global changes. The *Landsat 7* system enables researchers to monitor small-scale processes, such as deforestation, on a global scale. Satellites, such as *Landsat 7,* are accelerated to the speeds necessary for them to achieve orbit by large rockets, such as shuttle-booster rockets. Because the acceleration of any mass must follow Newton's second law of motion, $F_{net} = ma$, more force is required to launch a more massive satellite into orbit. Thus, the mass of a satellite is limited by the capability of the rocket used to launch it.

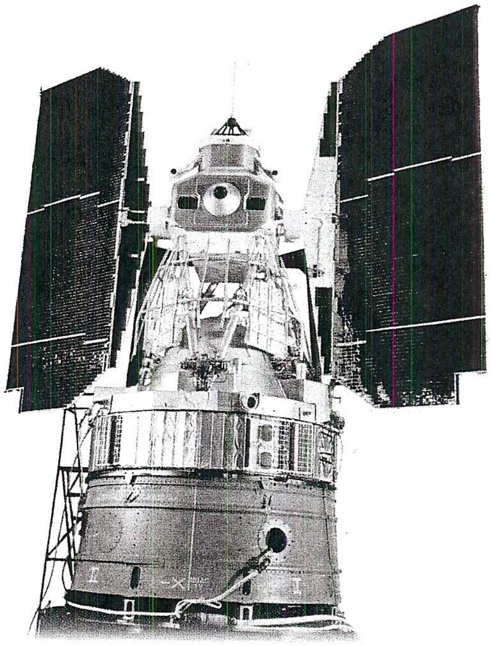


Figure 12 *Landsat 7* is capable of providing up to 532 images of Earth per day.

Free-Fall Acceleration

The acceleration of objects due to Earth's gravity can be found by using Newton's law of universal gravitation and his second law of motion. For a free-falling object of mass m, the following is true:

$$F = \frac{Gm_Em}{r^2} = ma, \text{ so } a = \frac{Gm_E}{r^2}$$

If you set $a = g_E$ and $r = r_E$ on Earth's surface, the following equation can be written:

$$g = \frac{Gm_E}{{r_E}^2}, \text{ thus, } m_E = \frac{g{r_E}^2}{G}$$

You saw above that $a = \frac{Gm_E}{r^2}$ for a free-falling object. Substitution of the above expression for m_E yields the following:

$$a = \frac{G\left(\frac{g{r_E}^2}{G}\right)}{r^2}$$

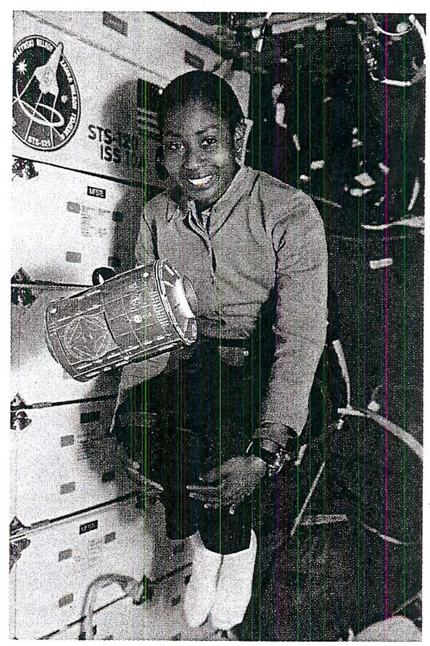
$$a = g\left(\frac{r_E}{r}\right)^2$$

On the surface of Earth, $r = r_E$ and so $a = g$. But, as you move farther from Earth's center, r becomes larger than r_E, and the free-fall acceleration is reduced according to this inverse square relationship. What happens to your mass as you move farther and farther from Earth's center?

Weight and weightlessness You may have seen photos similar to **Figure 13** in which astronauts are on a spacecraft in an environment often called zero-g or weightlessness. The spacecraft orbits about 400 km above Earth's surface. At that distance, $g = 8.7$ N/kg, only slightly less than that on Earth's surface. Earth's gravitational force is certainly not zero in the shuttle. In fact, gravity causes the shuttle to orbit Earth. Why, then, do the astronauts appear to have no weight?

Remember that you sense weight when something, such as the floor or your chair, exerts a contact force on you. But if you, your chair, and the floor all are accelerating toward Earth together, then no contact forces are exerted on you. Thus, your apparent weight is zero and you experience apparent weightlessness. Similarly, the astronauts experience apparent weightlessness as the shuttle and everything in it falls freely toward Earth.

Figure 13 Astronauts in orbit around Earth are in free fall because their spacecraft and everything in it is accelerating toward Earth along with the astronauts. That is, the floor is constantly falling from beneath their feet.



MiniLABs

WEIGHTLESS WATER

What are the effects of weightlessness in free fall?

WEIGHT IN FREE FALL

What is the effect of free fall on mass?

iLab Station

The Gravitational Field

Recall from studying motion that many common forces are contact forces. Friction is exerted where two objects touch; for example, the floor and your chair or desk push on you when you are in contact with them. Gravity, however, is different. It acts on an apple falling from a tree and on the Moon in orbit. In other words, gravity acts over a distance. It acts between objects that are not touching or that are not close together. Newton was puzzled by this concept. He wondered how the Sun could exert a force on planet Earth, which is hundreds of millions of kilometers away.

Field concept The answer to the puzzle arose from a study of magnetism. In the nineteenth century, Michael Faraday developed the concept of a field to explain how a magnet attracts objects. Later, the field concept was applied to gravity.

Any object with mass is surrounded by a gravitational field, which exerts a force that is directly proportional to the mass of the object and inversely proportional to the square of the distance from the object's center. Another object experiences a force due to the interaction between its mass and the gravitational field (g) at its location. The direction of g and the gravitational force is toward the center of the object producing the field. The gravitational field is expressed by the following equation.

GRAVITATIONAL FIELD

The gravitational field produced by an object is equal to the universal gravitational constant times the object's mass, divided by the square of the distance from the object's center.

$$g = \frac{Gm}{r^2}$$

Suppose the gravitational field is created by the Sun. Then a planet of mass m in the Sun's gravitational field has a force exerted on it that depends on its mass and the magnitude of the gravitational field at its location. That is, $\boldsymbol{F} = m\boldsymbol{g}$, toward the Sun. The force is caused by the interaction of the planet's mass with the gravitational field at its location, not with the Sun millions of kilometers away. To find the gravitational field caused by more than one object, calculate all gravitational fields and add them as vectors.

The gravitational field is measured by placing an object with a small mass (m) in the gravitational field and measuring the force ($\boldsymbol{F}$) on it. The gravitational field is calculated using $\boldsymbol{g} = \frac{\boldsymbol{F}}{m}$. The gravitational field is measured in units of newtons per kilogram (N/kg).

On Earth's surface, the strength of the gravitational field is 9.8 N/kg, and its direction is toward Earth's center. The field can be represented by a vector of length g pointing toward the center of the object producing the field. You can picture the gravitational field produced by Earth as a collection of vectors surrounding Earth and pointing toward it, as shown in **Figure 14.** The strength of Earth's gravitational field varies inversely with the square of the distance from Earth's center. Earth's gravitational field depends on Earth's mass but not on the mass of the object experiencing it.

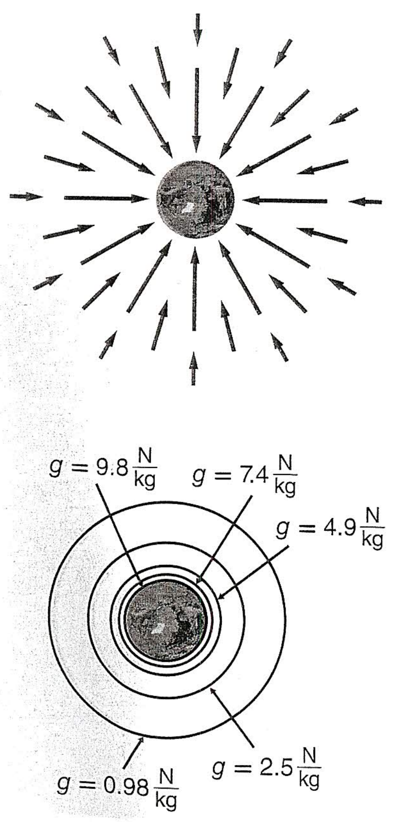


Figure 14 Earth's gravitational field can be represented by vectors pointing toward Earth's center. The decrease in $\boldsymbol{g}$'s magnitude follows an inverse-square relationship as the distance from Earth's center increases.

Explain why the value of $\boldsymbol{g}$ never reaches zero.

Two Kinds of Mass

You read that mass can be defined as the slope of a graph of force versus acceleration. That is, mass is equal to the net force exerted on an object divided by its acceleration. This kind of mass, related to the inertia of an object, is called inertial mass and is represented by the following equation.

INERTIAL MASS

Inertial mass is equal to the net force exerted on the object divided by the acceleration of the object.

$$m_{\text{inertial}} = \frac{F_{\text{net}}}{a}$$

Inertial mass You know that it is much easier to push an empty cardboard box across the floor than it is to push one that is full of books. The full box has greater inertial mass than the empty one. The **inertial mass** of an object is a measure of the object's resistance to any type of force. Inertial mass of an object is measured by exerting a force on the object and measuring the object's acceleration. The more inertial mass an object has, the less acceleration it undergoes as a result of a net force exerted on it.

Gravitational mass Newton's law of universal gravitation, $F = \frac{Gm_1m_2}{r^2}$, also involves mass—but a different kind of mass. Mass as used in the law of universal gravitation is a quantity that measures an object's response to gravitational force and is called **gravitational mass.** It can be measured by using a simple balance, such as the one shown in **Figure 15.** If you measure the magnitude of the attractive force exerted on an object by another object of mass m, at a distance r, then you can define the gravitational mass in the following way.

GRAVITATIONAL MASS

The gravitational mass of an object is equal to the distance between the centers of the objects squared, times the gravitational force, divided by the product of the universal gravitational constant, times the mass of the other object.

$$m_{\text{grav}} = \frac{r^2F_{\text{grav}}}{Gm}$$

How different are these two kinds of mass? Suppose you have a watermelon in the trunk of your car. If you accelerate the car forward, the watermelon will roll backward relative to the trunk. This is a result of its inertial mass—its resistance to acceleration. Now, suppose your car climbs a steep hill at a constant speed. The watermelon will again roll backward. But this time, it moves as a result of its gravitational mass. The watermelon is pulled downward toward Earth.

Newton made the claim that inertial mass and gravitational mass are equal in magnitude. This hypothesis is called the principle of equivalence. All investigations conducted so far have yielded data that support this principle. Most of the time we refer simply to the mass of an object. Albert Einstein also was intrigued by the principle of equivalence and made it a central point in his theory of gravity.

PhysicsLABs

HOW CAN YOU MEASURE MASS?

How is an inertial balance used to measure mass?

INERTIAL MASS AND GRAVITATIONAL MASS

How can you determine the relationship between inertial mass and gravitational mass?

iLab Station

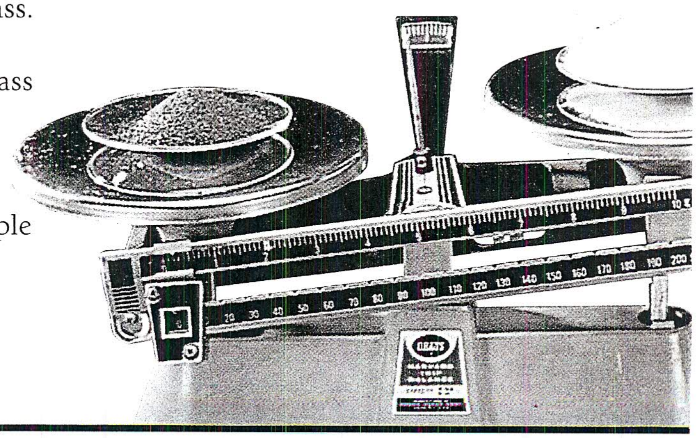


Figure 15 A simple balance is used to determine the gravitational mass of an object.

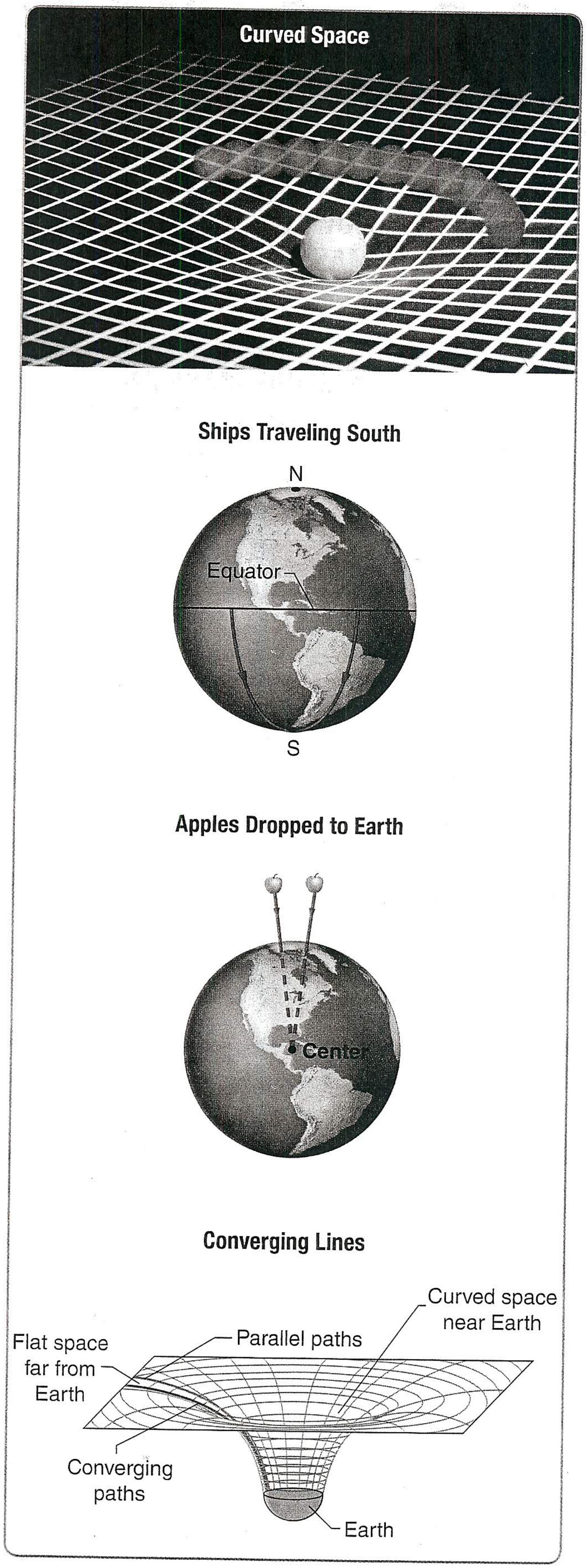


Figure 16 Visualizing how space is curved is difficult. Analogies can help you understand difficult concepts.

Einstein's Theory of Gravity

Newton's law of universal gravitation allows us to calculate the gravitational force that exists between two objects because of their masses. Newton was puzzled, however, as to how two objects could exert forces on each other if those two objects were millions of kilometers away from each other. Albert Einstein proposed that gravity is not a force but rather an effect of space itself. According to Einstein's explanation of gravity, mass changes the space around it. Mass causes space to be curved, and other bodies are accelerated because of the way they follow this curved space.

Curved space One way to picture how mass affects space is to model three-dimensional space as a large, two-dimensional sheet, as shown in the top part of **Figure 16.** The yellow ball on the sheet represents a massive object. The ball forms an indentation on the sheet. A red ball rolling across the sheet simulates the motion of an object in space. If the red ball moves near the sagging region of the sheet, it will be accelerated. In a similar way, Earth and the Sun are attracted to each other because of the way space is distorted by the two objects.

Ships traveling south The following is another analogy that might help you understand the curvature of space. Suppose you watch from space as two ships travel due south from the equator. At the equator, the ships are separated by 4000 km. As they approach the South Pole, the distance decreases to 1 km. To the sailors, their paths are straight lines, but because of Earth's curvature, they travel in a curve, as viewed far from Earth's surface, as in **Figure 16.**

Apples dropped to Earth Consider a similar motion. Two apples are dropped to Earth, initially traveling in parallel paths, as in **Figure 16.** As they approach Earth, they are pulled toward Earth's center. Their paths converge.

Converging lines This convergence can be attributed to the curvature of space near Earth. Far from any massive object, such as a planet or star, space is flat, and parallel lines remain parallel. Then they begin to converge. In flat space, the parallel lines would remain parallel. In curved space, the lines converge.

Einstein's theory or explanation, called the general theory of relativity, makes many predictions about how massive objects affect one another. In every test conducted to date, Einstein's theory has been shown to give the correct results.

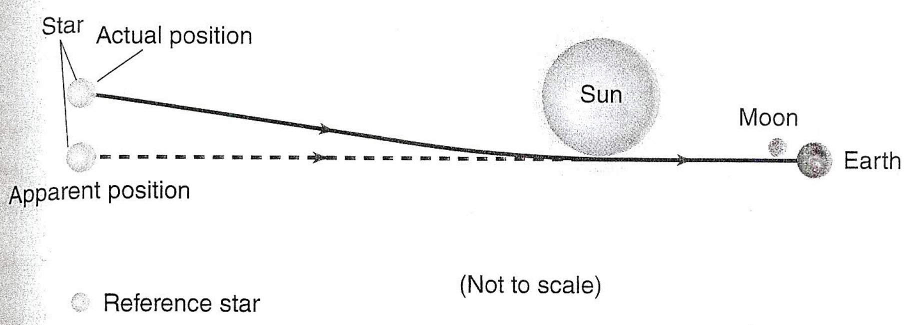


Figure 17 Light is bent around massive objects in space, altering their apparent position.

Describe how this effect contradicts your experience of light's behavior.

Deflection of light Einstein's theory predicts that massive objects deflect and bend light. Light follows the curvature of space around the massive object and is deflected, as shown in **Figure 17.** In 1919, during an eclipse of the Sun, astronomers found that light from distant stars that passed near the Sun was deflected an amount that agreed with Einstein's predictions.

Another result of general relativity is the effect of gravity on light from extremely massive objects. If an object is massive and dense enough, the light leaving it is totally bent back to the object. No light ever escapes the object. Objects such as these, called black holes, have been identified as a result of their effects on nearby stars. Black holes have been detected through the radiation produced when matter is pulled into them.

While Einstein's theory provides very accurate predictions of gravity's effects, it is still incomplete. It does not explain the origin of mass or how mass curves space. Physicists are working to understand the deeper meaning of gravity and the origin of mass itself.

SECTION 2 REVIEW

Section Self-Check Check your understanding.

18. **MAIN**IDEA The Moon is 3.84×10^5 km from Earth's center and Earth is 14.96×10^7 km from the Sun's center. The masses of Earth and the Sun are 5.97×10^{24} kg and 1.99×10^{30} kg, respectively. During a full moon, the Sun, Earth, and the Moon are in line with each other, as shown in **Figure 18.**

a. Find the ratio of the gravitational fields due to Earth and the Sun at the center of the Moon.

b. What is the net gravitational field due to the Sun and Earth at the center of the Moon?

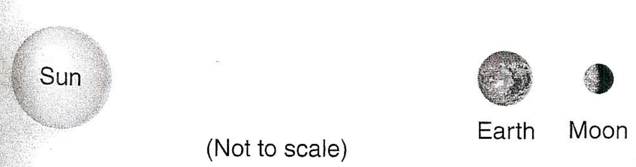


Figure 18

19. **Apparent Weightlessness** Chairs in an orbiting spacecraft are weightless. If you were on board such a spacecraft and you were barefoot, would you stub your toe if you kicked a chair? Explain.

20. **Gravitational Field** The mass of the Moon is 7.3×10^{22} kg and its radius is 1785 km. What is the strength of the gravitational field on the surface of the Moon?

21. **Orbital Period and Speed** Two satellites are in circular orbits about Earth. One is 150 km above the surface, the other is 160 km.

a. Which satellite has the larger orbital period?

b. Which has the greater speed?

22. **Theories and Laws** Why is Einstein's description of gravity called a theory, while Newton's is a law?

23. **Astronaut** What would be the strength of Earth's gravitational field at a point where an 80.0-kg astronaut would experience a 25.0 percent reduction in weight?

24. **A Satellite's Mass** When the first artificial satellite was launched into orbit by the former Soviet Union in 1957, U.S. president Dwight D. Eisenhower asked scientists to calculate the mass of the satellite. Would they have been able to make this calculation? Explain.

25. **Critical Thinking** It is easier to launch a satellite from Earth into an orbit that circles eastward than it is to launch one that circles westward. Explain.

CHAPTER 7 STUDY GUIDE

BIGIDEA Gravity is an attractive field force that acts between objects with mass.

SECTION 1 Planetary Motion and Gravitation

VOCABULARY

- **Kepler's first law** *(p. 179)*
- **Kepler's second law** *(p. 179)*
- **Kepler's third law** *(p. 180)*
- **gravitational force** *(p. 182)*
- **law of universal gravitation** *(p. 182)*

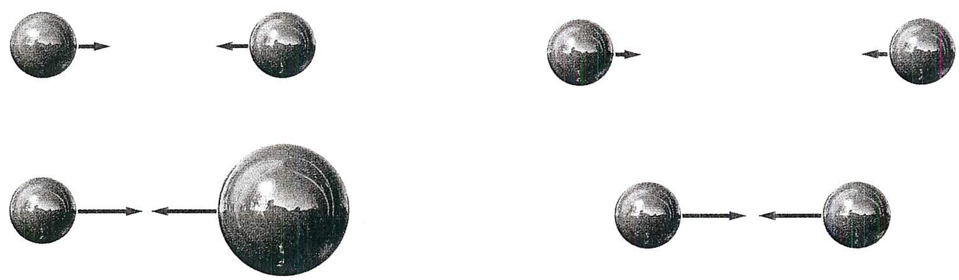
MAINIDEA The gravitational force between two objects is proportional to the product of their masses divided by the square of the distance between them.

- Kepler's first law states that planets move in elliptical orbits, with the Sun at one focus, and Kepler's second law states that an imaginary line from the Sun to a planet sweeps out equal areas in equal times. Kepler's third law states that the square of the ratio of the periods of any two planets is equal to the cube of the ratio of their distances from the Sun.

$$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$$

- Newton's law of universal gravitation can be used to rewrite Kepler's third law to relate the radius and period of a planet to the mass of the Sun. Newton's law of universal gravitation states that the gravitational force between any two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers. The force is attractive and along a line connecting the centers of the masses.

$$F = \frac{Gm_1m_2}{r^2}$$



- Cavendish's investigation determined the value of G, confirmed Newton's prediction that a gravitational force exists between two objects, and helped calculate the mass of Earth.

SECTION 2 Using the Law of Universal Gravitation

VOCABULARY

- **inertial mass** *(p. 191)*
- **gravitational mass** *(p. 191)*

MAINIDEA All objects are surrounded by a gravitational field that affects the motions of other objects.

- The speed and period of a satellite in circular orbit describe orbital motion. Orbital speed and period for any object in orbit around another are calculated with Newton's second law.
- Gravitational mass and inertial mass are two essentially different concepts. The gravitational and inertial masses of an object, however, are numerically equal.
- All objects have gravitational fields surrounding them. Any object within a gravitational field experiences a gravitational force exerted on it by the gravitational field. Einstein's general theory of relativity explains gravitational force as a property of space itself.

Games and Multilingual eGlossary

Vocabulary Practice