LEARN BY DRAWING GRAPHICAL RELATIONSHIP BETWEEN ELECTRIC FIELD LINES AND EQUIPOTENTIALS

Because it takes no work to move a charge along an equipotential surface, such surfaces must be perpendicular to the electric field lines. Also, the electric field has a magnitude equal to the change in potential per unit distance (V/m) and points in the direction in which the potential decreases most rapidly. These facts can be used to construct equipotentials if we know the field pattern. The reverse is also true: Given the equipotentials, the electric field lines can be constructed. Furthermore, if the potential (in volts) associated with each equipotential is known, the strength and direction of the field can be estimated from the rate at which the potential changes with distance (Eq. 16.8).

A couple of examples should provide a graphical insight into the connection between equipotential surfaces and their associated electric fields. Consider Fig. 1, in which you are given the electric field lines and want to determine the shape of the equipotentials. Pick any point, such as A, and begin moving at right angles to the field lines. Keep moving so as to maintain this perpendicular orientation to the lines. Between lines you may have to approximate, but plan ahead to the next field line so it is crossed at a right angle. To find another equipotential, start at another point, such as B, and proceed the same way. Sketch as many equipotentials as you need to map the area of interest. The figure shows the result of sketching four equipotentials, from A (at the highest potential—can you tell why?) to D (at the lowest potential).

Now suppose you are given the equipotentials instead of the field lines (Fig. 2). The electric field lines point in the direction of decreasing V and are perpendicular to the equipotential surfaces. Thus, to map the field, start at any point, and move in such a way that your path intersects each equipotential surface at a right angle. The resulting field line is shown in Fig. 2, beginning at point A. Starting at points B, C, and D provides additional field lines that suggest the complete electric field pattern; you need only add the arrows in the direction of decreasing potential.

Lastly, suppose you want to estimate the magnitude of $\vec{\mathbf{E}}$ at some point P (Fig. 3), knowing the values of the equipotentials 1.0 cm on either side of it. From this, you know that the field points roughly from A to B (why?) and its approximate magnitude would be

$$E = \left|\frac{\Delta V}{\Delta x}\right|_{max} = \frac{(1000\text{ V} - 950\text{ V})}{2.0 \times 10^{-2}\text{ m}} = 2.5 \times 10^{3}\text{ V/m}$$

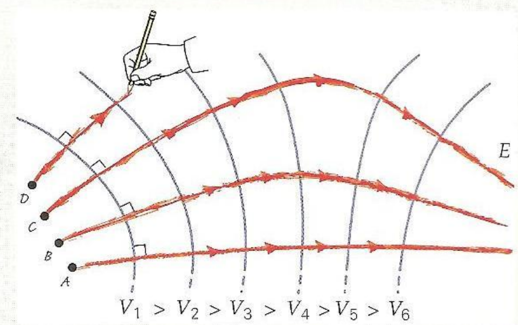

FIGURE 2 Mapping the electric field from equipotentials Start at a convenient point, and trace a line that crosses each equipotential at a right angle. Repeat the process as often as needed to reveal the field pattern, adding arrows to indicate the direction of the field lines from high to low potential. In going from one potential to the next, plan ahead so that each succeeding equipotential is also crossed at right angles.

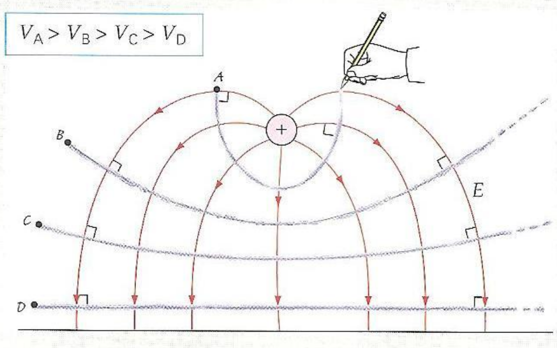

FIGURE 1 Sketching equipotentials from electric field lines If you know the electric field pattern, pick a point in the region of interest and move so that your path is always perpendicular to the next field line. Keep your path as smooth as possible, planning ahead so that each succeeding field line is also crossed at right angles. To map a surface with a higher (or lower) potential, move in the opposite (or the same) direction as the electric field and repeat the process. Here, $V_A > V_B$, and so on.

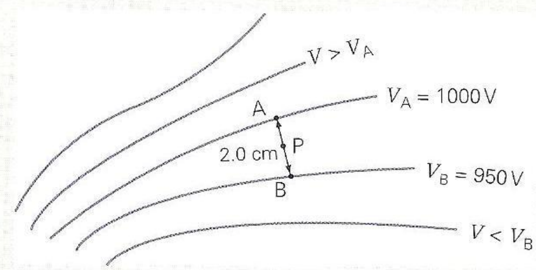

FIGURE 3 Estimating the magnitude of the electric field The magnitude of the potential change per meter at any point gives the strength of the electric field at that point.