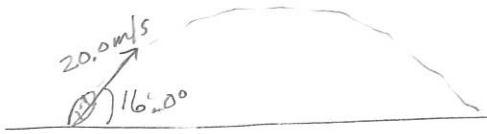
Problem Set 2 — Projectiles

① 

$v_x = \cos 16(20) = 19.225\ m/s$

$v_y = \sin 16(20) = 5.513\ m/s$

A) $v_{y1} = +5.513\ m/s$, $v_{y2} = -5.513\ m/s$, $a_y = g = -9.80\ m/s^2$

$$t = \frac{v_{y2} - v_{y1}}{g} = \frac{-5.513\ m/s - 5.513\ m/s}{-9.80\ m/s^2} = 1.125\ sec$$

$\boxed{t = 1.12\ sec}$

B) $v_{y1} = +5.513\ m/s$, $v_{y2} = 0$, $g = -9.80\ m/s^2$

$$d = \frac{v_{y2}^2 - v_{y1}^2}{2g} = \frac{0^2 - (5.513)^2}{2(-9.80)} = 1.55\ m$$

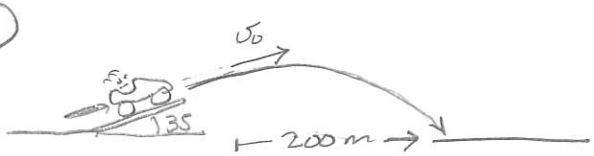
$\boxed{d_{y\,max} = 1.55\ m}$

C) $v_x = 19.225\ m/s$, $t = 1.125\ sec$

$$d_x = v_x t = (19.225)(1.125) = 21.628\ m$$

$\boxed{d_x = 21.6\ m}$

D) $v_0 = v_f$ $\boxed{v_f = 20.0\ m/s\ \ 16.0°\ from\ Horizontal}$

② 

$\Theta = 35°$, $d_x = 200\ m$

$v_x = \cos 35 v_0$

$v_y = \sin 35 v_0$

$d_y = 0$

$d_x = 200\ m$

$d_x = v_x t$ $\quad t = \dfrac{d_x}{\cos 35 v_0}$

$d_y = v_{y1} t + \frac{1}{2} g t^2$

$$d_y = \sin 35 v_0 \left(\frac{d_x}{\cos 30 v_0}\right) + \frac{1}{2} g \left(\frac{d_x}{\cos 30 v_0}\right)^2$$

$$0 = \tan 35 (200) + -4.9 \left(\frac{(200)^2}{(\cos 30)^2 v_0^2}\right)$$

$$0 = 140 + -\frac{29209.7}{v_0^2}$$

$$v_0^2 = 2086.4$$

$\boxed{v_0 = 45.7\ m/s\ @\ 35\ from\ Horizontal}$

③ 

$v_x = \cos 30 (13.0) = 11.26\ m/s$

$v_y = \sin 30 (13.0) = 6.5\ m/s$

A) $\boxed{v_x = 11.3\ m/s}$

B) $\boxed{v_y = 6.50\ m/s}$

C) $$t = \frac{v_{y2} - v_{y1}}{g} = \frac{-6.50 - 6.50}{-9.80} = 1.326\ sec$$ $\boxed{t = 1.33\ sec}$

D) $v_{y1} = 6.50\ m/s$, $v_{y2} = 0$, $a_y = g = -9.80\ m/s^2$

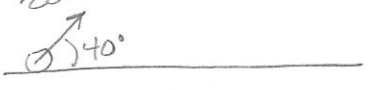
$$d_y = \frac{v_{y2}^2 - v_{y1}^2}{2g} = \frac{0 - (6.50)^2}{2(-9.8)} = 2.155\ m$$ $\boxed{d_{y\,max} = 2.16\ m}$

E) $v_x = 11.26\ m/s$, $t = 1.326\ s$

$$d_x = v_x t = (11.26\ m/s)(1.326) = 14.93\ m$$

$\boxed{d_x = 14.9\ m}$

④



$v_x = \cos 40(20) = 15.321$

$v_{y_1} = \sin 40(20) \quad 12.856$

A) $t = \frac{v_{2y} - v_{1y}}{g} = \frac{-12.856 - 12.856}{-9.8} = 2.623$

$\boxed{t = 2.62 \text{ sec}}$

B) $d_y = \frac{v_{y_2}^2 - v_{y_1}^2}{2g} = \frac{0 - (12.856)^2}{2(-9.8)} = 8.4325$

$\boxed{d_{y\max} = 8.43\text{m}}$

C) $d_x = v_x t = (15.321)(2.623) = 40.187$

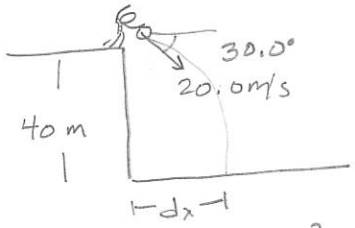
$\boxed{d_x = 40.2\text{m}}$

D) $v_{x_2} = 15.321$

$v_{y_2} = -12.856$

$\sqrt{15.321^2 + (12.856)^2} = 20.0 \text{ m/s}$ @ $40.0°$ from horizontal

⑤



$v_x = \cos 30(20\text{m}) = 17.32 \text{ m/s}$

$v_y = \sin 30(20\text{m}) = 10.0 \text{ m/s}$

$d_y = -40\text{m}$

$d_y = v_{y_1} t + \frac{1}{2} g t^2$

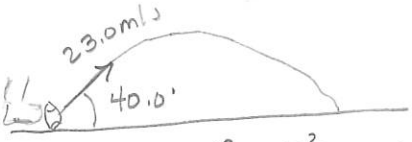
$-40 = -10t + -4.9t^2$

$t = 1.82 \text{ sec}$

$d_x = v_x t = (17.32)(1.818) = 31.494$

$\boxed{d_x = 31.5\text{m}}$

⑥



$v_{x_1} = \cos 40(23.0) = 17.619$

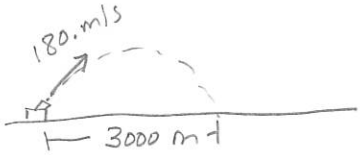
$v_{y_1} = \sin 40(23.0) = 14.784$

A) $d_y = \frac{v_{2y}^2 - v_{1y}^2}{2g} = \frac{0 - (14.784)^2}{2(-9.8)} = 11.15153$ $\boxed{d_y = 11.2\text{m}}$

B) $d_x = v_x t = v_x\left(\frac{v_{y_2} - v_{y_1}}{g}\right) = 17.619\left(\frac{-14.784 - 14.784}{-9.8}\right) = 53.159$

$\boxed{d_x = 53.2\text{m}}$

⑦



$v_x = \cos\theta(180)$

$v_y = \sin\theta(180)$

$d_x = 3000\text{m}$

$t = \frac{-2v_y}{g} = \frac{-2\sin\theta(180)}{-9.8}$

$d_x = v_x t = \frac{\cos\theta v_0 2\sin\theta v_0}{g}$

$d_x = \frac{\sin(2\theta) v_0^2}{g}$

$2\theta = \sin^{-1}\left(\frac{d_x g}{v_0^2}\right) = \sin^{-1}\frac{(3000)(9.8)}{(180)^2}$

$2\theta = 65.149 \qquad \theta = 32.575°$

A) $\boxed{\theta = 32.6°}$

B) @ $45°$

$v_x = \cos 45°(180) = 127.28 \text{ m/s}$

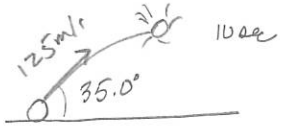
$v_y = \sin 45(180) = 127.28 \text{ m/s}$

$d_x = v_x t = v_x \cdot \frac{2v_y}{g} = 127.28 \frac{-2(127.28)}{-9.8}$

$\boxed{d_{x\max} = 3306\text{m}}$

So it can NOT hit @ 3500m

(8)



$v_x = \cos 35(125) = 102.39$

$v_y = \sin 35(125) \quad 71.697$

$t = 10 \text{ sec}$

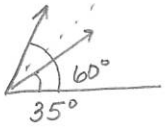
$d_x = v_x t = (102.39)(10) = 1023.9 \text{ m}$

$d_y = v_y t + \frac{1}{2} g t^2 = (71.697)(10) + \frac{1}{2}(-9.8)(10)^2$

$d_y = 226.97$

$\vec{x} = 1024 \text{ m} \quad \vec{y} \; 227 \text{ m}$

(9)



350 WOULD GO FORTHER, CLOSER TO THE OPTIMUM 45° ANGLE

$d_x = v_x t \qquad t = \frac{2 \sin\theta v_0}{g} \qquad v_x = \cos\theta v_0$

$d_x = \frac{\sin(2\theta) v_0^2}{g}$

@ 35°: $\frac{(\sin(2(35)))(v_0^2)}{g}$

60°: $\frac{\sin 2(60)\, v_0^2}{g}$

$= \frac{\sin 70}{\sin 120} = 1.08$ — 8% further

(10)

maximum range = $\theta = 45°$

$v_x = \cos 45°(36) = 25.46$

$v_y = \sin 45(36) = 25.46$

$g_m = \frac{1}{6} g_E = \frac{1}{6}(9.8) = 1.63 \text{ m/s}^2$

$d_x = v_x t = v_x\left(\frac{-2v_y}{g_m}\right)$

$= 25.46 \; \frac{-2(25.46)}{-1.63 \text{ m/s}^2}$

$d_x = 793.7 \text{ m}$

$d_x = 794 \text{ m}$