Circular Motion Problems III

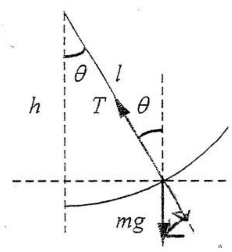
1. To keep the water from coming out of the bucket, we have to "use up" the gravitational force in centripetal force. $mg = F_c = m r \omega^2$, $\Rightarrow$ $\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{9.80\ \text{m/s}^2}{0.95\ \text{m}}} = \boxed{3.2\ \text{rad/s}}$.

$F_c = mg$

$\frac{mv^2}{r} = mg$

$v = \sqrt{rg} = \sqrt{(.95\text{m})(9.8\text{m/s}^2)} = 3.05\ \text{m/s}$

2. The centripetal force is provided by the combination of



$T - mg\cos\theta = m\frac{v^2}{r}$, where $\cos\theta = \frac{h}{l}$ and v could be found from energy conservation. When the girl is h below the original position,

we have $\frac{1}{2}m(0)^2 + mgh = \frac{1}{2}mv^2 + mg(0)$, so $v^2 = 2gh$.

Therefore $T = mg\cos\theta + m\frac{v^2}{r} = \frac{mgh}{l} + \frac{m(2gh)}{l} = \frac{3mgh}{l}$.

(a) $h = 0$, so $T = \boxed{0}$.

(b) $h = 12\ m - 5.0\ \text{m} = 7.0\ \text{m}$, so $T = \frac{3(60\ \text{kg})(9.80\ \text{m/s}^2)(7.0\ \text{m})}{10\ \text{m}} = \boxed{1.2\times10^3\ \text{N}}$.

(c) $h = 12\ \text{m} - 2.0\ \text{m} = 10\ \text{m}$, so $T = \frac{3(60\ \text{kg})(9.80\ \text{m/s}^2)(10\ \text{m})}{10\ \text{m}} = \boxed{1.8\times10^3\ \text{N}}$.

3. (a) Use the diagram and the results in Exercise 7.108.

In the tangential direction (perpendicular to the rope): $F_{\text{net}} = -mg\sin\theta = ma$,

so $a = g\sin\theta = (9.80\ \text{m/s}^2)\sin 30° = \boxed{4.9\ \text{m/s}^2}$.

(b) $a_c = \frac{v^2}{r} = \frac{(8.5\ \text{m/s})^2}{10\ \text{m}} = 7.23\ \text{m/s}^2$. So the magnitude of the vector sum of the tangential and centripetal accelerations is $a = \sqrt{(4.9\ \text{m/s}^2)^2 + (7.23\ \text{m/s}^2)^2} = \boxed{8.7\ \text{m/s}^2}$.

(c) Assume she starts at h' above the pond surface (with zero velocity).

When $\theta = 30°$, $h = (10\ \text{m})\cos 30° = 8.66$ m or she is 12 m − 8.66 m = 3.34 m above the pond surface.

From energy conservation: $\frac{1}{2}m(8.5\ \text{m/s})^2 + mg(3.34\ \text{m}) = \frac{1}{2}m(0)^2 + mgh'$,

so $h' = \frac{(8.5\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} + 3.34\ \text{m} = \boxed{7.0\ \text{m}}$.

5. (a) In the vertical direction: $N \cos\theta - mg = 0$, ☞ $N = \dfrac{mg}{\cos\theta}$.

In the horizontal direction: $F_c = N \sin\theta = \dfrac{mg}{\cos\theta}\sin\theta = mg\tan\theta = m\dfrac{v^2}{r}$, so $\tan\theta = \dfrac{v^2}{gr}$.

(b) Since mass is not in the result in part (a), the angle is $\boxed{\text{independent of mass}}$.

(c) If there is friction, it will point along the inclined plane,

so the contribution from friction to centripetal force is $f_s\cos\theta = (\mu_s N)\cos\theta = \mu_s \dfrac{mg}{\cos\theta} = \mu_s mg$.

Therefore $mg\tan\theta + \mu_s mg = m\dfrac{v^2}{r}$, ☞ $\boxed{\tan\theta = \dfrac{v^2}{gr} - \mu_s}$.

As expected, the angle does not need to be as big as in part (a) when there is friction.

6. The normal force N provides centripetal force. $N = m\dfrac{v^2}{r}$, ☞ $v = \sqrt{\dfrac{Nr}{m}}$. Eq. (1)

Also $f_s = \mu_s N = mg$ for the rider not to slide down the wall, so $N = \dfrac{mg}{\mu_s}$. Eq. (2)

Substituting Eq. (2) into Eq. (1), we have

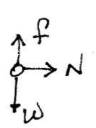
$$v = \sqrt{\frac{gr}{\mu_s}} = \sqrt{\frac{(9.80\ \text{m/s}^2)(15\ \text{m})}{1.1}} = \boxed{12\ \text{m/s} = 43\ \text{km/h} = 27\ \text{mi/h}}.$$

(b) $\boxed{\text{No}}$ because the result in (a) is independent of m.

4. a) $v = 2\pi r f = 2\pi(2.0\,\text{m})(1.1\,\text{Hz}) = 13.82\,\text{m/s} = 14\,\text{m/s}$

b) $a_c = \dfrac{v^2}{r} = \dfrac{(13.82\,\text{m/s})^2}{2.0\,\text{m}} = 95.54\,\text{m/s}^2 = 95\,\text{m/s}^2$

c) THE WALL'S NORMAL FORCE ON THE RIDERS BACK.

D) 

$F_c = N$

$\dfrac{mv^2}{r} = N$

$f = mg$

$\mu N = mg$

$\mu \dfrac{mv^2}{r} = mg$

$\mu = rg/v^2$

$\mu = \dfrac{(2.0\,\text{m})(9.8\,\text{m/s}^2)}{(13.82\,\text{m/s})^2} = 0.103$