

Dimensional Analysis

Quantities such as length, speed, and area are called *dimensional quantities*. A measured dimensional quantity has a numerical value that depends upon the system of units used. For example, a given area can be stated as either 1 square meter or 10 000 square centimeters. When making a measurement, the most convenient unit is used.

When substituting values into an equation in physics, you must state the units as well as the numerical values. Including units in your calculations helps you keep units consistent throughout and assures you that your answer will be dimensionally correct.

You may also use the units or dimensions of your measurements to check the correctness of your equation.

Example

Dimensionally Incorrect Equation

$$\text{velocity} \stackrel{?}{=} (\text{distance}) (\text{time})$$

$$\frac{\text{m}}{\text{s}} \stackrel{?}{=} \frac{\text{m}|\text{s}}{1}$$

$$\frac{\text{m}}{\text{s}} \neq \text{m} \cdot \text{s}$$

Note that the final units on the right do not equal those on the left. By inspecting the dimensions, you should be immediately aware that the equation is not correctly written.

Example

Dimensionally Correct Equation

$$\text{mass} \stackrel{?}{=} (\text{density}) (\text{volume})$$

$$\text{kg} \stackrel{?}{=} \frac{\text{kg}|\text{m}^3}{\text{m}^3}$$

$$\text{kg} = \text{kg}$$

Note that some of the units on the right side of the equation cancel out. The final dimensions on both sides are the same. The equation is dimensionally correct.

Problems

Use the method described above to determine if the following equations are correctly written. The proper units for the variables are listed below.

d (distance)-meters

v (velocity)-meters/second

t (time)-seconds

a (acceleration)-meters/(second)²

v_i = initial velocity; v_f = final velocity; \bar{v} = average velocity

1. $v_f \stackrel{?}{=} v_i t + a$

6. If r is in meters

g is in meters/s²

x is in kg/meters³

n is in kg/meter s

does $v = \frac{2 r^2 g x}{9\pi n}$?

2. $\bar{v} \stackrel{?}{=} \frac{v_i + v_f}{2}$

3. $d \stackrel{?}{=} \sqrt{v_i t^2 + \frac{1}{2} a t}$

4. $v_f \stackrel{?}{=} v_i^2 + 2 a d^2$

5. $d \stackrel{?}{=} \frac{1}{2} v_i t + a t$